Pre Calculus

Date:

Items Needed: .Book, graph paper

Objective: The students will be able to evaluate inverse trig functions and also compositions of trig functions.

PA Common Core: cc.2.2.hs.c.8, cc.2.2.hs.c.4

Lesson:

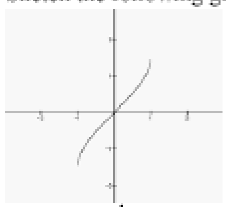
- Remember how to determine whether a function has an inverse? State that because the sine function does not pass the Horizontal Line Test, we must restrict its domain in order for its inverse to be a function. We restrict the domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
-
- Graph $y = \sin x$. Look at the x interval between the values of $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ or graph it between that interval.
- What can you tell me about the function.
  1. Increasing
  2. Takes on its full range of *y* values.
  3. Is one to one
- Inverse sine function is denoted by $y = \sin^{-1} x$ or $y = \arcsin x$. Arcsin x means the angle (or arc) whose sine is x. Remember that $\sin^{-1} x$ denotes the inverse sine function rather than $y = \frac{1}{\sin x}$.
- Graph $y = \sin^{-1} x$.
- Refer to the definition of the inverse sine function of p. 315.
- Point out the domain and range.
- Do example 1.

- We can do the same thing with the cosine and tangent functions.
- Look at the limitations we have with the domain and range of the inverse functions, refer to p. 317.

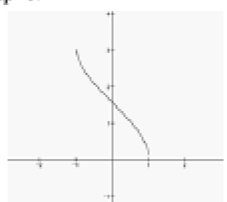
State the following definitions.

Function	*Domain*	*Range*
$y = \sin^{-1}x \Leftrightarrow \sin y = x$	[-1, 1]	[-π/2, π/2]
$y = \cos^{-1}x \Leftrightarrow \cos y = x$	[-1, 1]	[0, π]
$y = \tan^{-1}x \Leftrightarrow \tan y = x$	(-∞, ∞)	(-π/2, π/2)

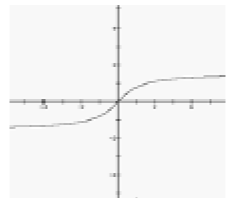
Sketch the following graphs.



$y = \sin^{-1} x$



$y = \cos^{-1}x$



$y = \tan^{-1}x$

- Look at example 3, and these examples

Example 2. Evaluate the following.

a) $\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

b) $\tan^{-1} 1 = \pi/4$

- Talk about using a calculator. Look at example 4.
- Don't forget about the domains.
- If you had set the calculator to degree mode, the displays would have been in degrees rather than radians. This convention is peculiar to calculators. By definition, the values of inverse trigonometric functions are always in radians.

- Look at transformations of arc functions, example 5. Note that they have the same characteristics of the transformations with the regular parent functions.

- Look at these properties

Review, from Section 1.8, that for inverse functions, $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

State the **inverse properties of trigonometric functions**.

1. If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then $\sin(\sin^{-1} x) = x$ and $\sin^{-1}(\sin y) = y$.
2. If $-1 \le x \le 1$ and $0 \le y \le \pi$, then $\cos(\cos^{-1} x) = x$ and $\cos^{-1}(\cos y) = y$.
3. If $-\pi/2 < y < \pi/2$, then $\tan(\tan^{-1} x) = x$ and $\tan^{-1}(\tan y) = y$.

- Do example 6. Look at the note on part b that I put in the book.
- Do example 7 using right triangles to set them up. Use the equivalent equations and the SOHCOHTOA ratios to set up the right triangle. If you need more examples do 5 from below.

Example 4. Evaluate the following.

a) sin(arcsin 0.12) = 0.12

b) arctan(tan $5\pi/6$) = arctan[tan $(-\pi/6)$] = $-\pi/6$

c) cos(arccos 6). 6 is not in the domain of the inverse cosine.

Example 5. Find the exact value.

a) $\cos(\sin^{-1} 3/5)$
Let $u = \sin^{-1} 3/5$ and draw a right triangle showing u, with the side opposite u having length 3 and the hypotenuse having length 5. Use the Pythagorean Theorem to label the other side 4. Then $\cos(\sin^{-1} 3/5) = \cos u = 4/5$.

b) $\sin[\tan^{-1}(-1/2)]$
Let $u = \tan^{-1}(-1/2)$ and draw a right triangle showing u, with the side opposite u having length -1 and the side adjacent having length 2. Use the Pythagorean Theorem to label the hypotenuse $\sqrt{5}$. Then $\sin[\tan^{-1}(-1/2)] = \sin u = \frac{-1}{\sqrt{5}}$.

Example 6. Write $\sin(\cos^{-1} x)$ as an algebraic expression in x.
Let $u = \cos^{-1} x$ and draw a right triangle showing u, with the side adjacent u having length x and the hypotenuse having length 1. Use the Pythagorean Theorem to label the other side $\sqrt{1-x^2}$.
Then $\sin(\cos^{-1} x) = \sin u = \sqrt{1-x^2}/1 = \sqrt{1-x^2}$.

- You can also use the right triangle to help solve composite functions that won't work out to a radian number, that is, they will still contain a variable x in them.
- Do example 8.

Homework Parts 2, 3, & 4 all have examples chosen to do before you assign the problems.

Assignment: .Have students do 1-4, 6, 9, 11, 12, 21-30(3), p. 322.
Have students do 35, 38, 42, p. 323.
Have students do 46, 48, 50, 52, 54-64(even), p. 323.
Have students do 67, 68, 69, 72, 73, 76, 77, 82, p. 323.
Have students do 99, p. 324.

Evaluation: (Could be from any one/several of the following)
Responses from classroom questions
Results of classroom sample problems
Homework responses
Check answer with Calculator
End of the section exam

Enrichment: